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Synthesis of RC active networks using transistors for the active element

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SYNTHESIS OF RC ACTIVE NETWORKS USING TRANSISTORS FOR THE ACTIVE ELEMENT

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I ABSTRACT

The basic objective of this work is to obtain a practical method for synthesizing RC Active Filters using transistors as the active element. After deriving a procedure for realizing any second order function, a transfer function with a high pass Butterworth characteristic is designed and constructed. The response of this filter is compared with one constructed according to the Sallen and Key vacuum-tube design procedure and with another which follows the Linvill and Horowitz design methods.

The sensitivity of the high pass filter is calculated and a method for minimizing it is obtained. The sensitivity is compared to that of the Linvill filter which uses a negative-impedance converter.

II INTRODUCTION

Although the tests to prove a function is positive-real are often long and numerically involved, the attribution of this property to a function is very desirable. It has been proved[1] that, if a positive-real (p-r) function is considered as a driving-point impedance, this function can always be synthesized using passive elements only. Various synthesis techniques for known positive-real functions are described in the above reference.

Although positive-reality is a powerful tool for use in network synthesis, it does not solve all the problems. The most general realization of a passive network, at best involves only RLC components (no transformers). However, if the problem under consideration involves a low-frequency range, the inductance values obtained by normal synthesis are rather large. Unfortunately, to realize these inductances with low losses means having inductors of large physical dimensions and weight, not to mention high cost.

A more desirable realization would be one involving only resistors and capacitors. It is well known, however, that an impedance function which can be realized by only RC components is a special type of function.[1] The poles and zeroes of such a function must lie on the negative real axis (of the complex frequency plane) only, and must have a specific orientation. This class of functions is very limited of course, therefore, other methods of realizing the totality of p-r functions using only RC components was sought.

An important advance came with the discovery that any polynomial can be separated into the difference of two polynomials, each of which has only negative real roots.

$$P(s) = Q(s) - KR(s)$$

Actually, $Q(s)$ and $R(s)$ are of the special form of polynomial which can be realized by RC networks. This now introduces the possibility of using an active element to realize the $-K$ term.

It was pointed out by Horowitz[2] that the process of realization by RC active networks is one of addition and subtraction of polynomials. This led to two classifications of the RC active network synthesis process. The first uses polynomial addition techniques and involves an amplifier with a selective feedback loop, and the second uses subtraction by means of various forms of negative-impedance converters.[3] (Linvill[4] presented this synthesis process originally and Horowitz[5] modified it to give optimum pole locations.) Although the above classification is not totally accurate, it is sufficient.

The basic problem is to realize complex conjugate roots with RC components. It will be sufficient to consider only a second order function of the quadratic form

$$s^2 + ds + 1$$

This follows because polynomials of higher degree can be separated into a product of first and second order functions. The first order functions are realizable with passive elements, therefore it is necessary to realize only the complex roots of the above equation by active elements.

A root locus of this equation is shown in Figure 1.

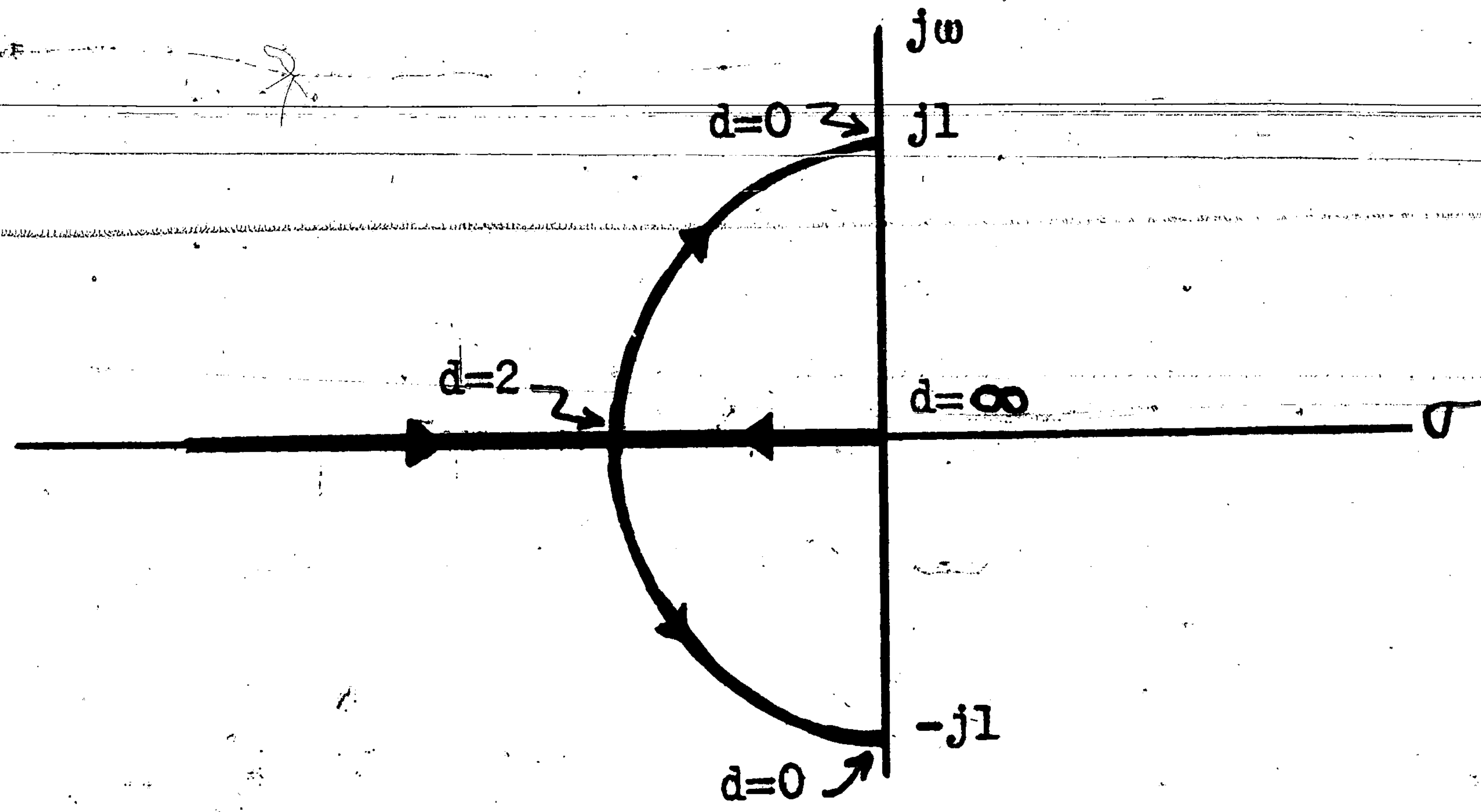


FIGURE 1

Notice that if d is greater than two ($d \geq 2$) the roots lie on the negative real axis and the function can be realized with RC elements only. Therefore, $d < 2$ is the only case which must be considered.

Sallen and Key[6] have developed a synthesis method to realize various values of d. The technique employs a vacuum-tube amplifier and one of a series of feedback networks with which any voltage transfer function of the form

$$G(s) = \frac{N(s)}{s^2 + ds + 1}$$

can be realized. The numerator N(s) may be a constant, first, or second order function, and is obtained by selecting the proper circuit. Therefore, given a G(s) to be realized, it is necessary to pick one of the circuits shown in the article and evaluate the components to satisfy the required d value. The exact procedure will be indicated later.

The Sallen and Key analysis is based on the assumption that the active element is an ideal voltage amplifier. Since a vacuum-tube amplifier, which is generally a good approximation to an ideal voltage amplifier is used, the assumption seems to be a valid one.

It appears reasonable to explore the possibility of realizing the same transfer functions using transistors as the active element rather than tubes. One of the objectives of this work is to develop a synthesis procedure using transistors. The approach will closely parallel that of Sallen and Key in the hope that a similar design procedure can be obtained.

The practicability of the technique will then be explored by construction and testing of a high pass filter. The results of the test will be compared to the same filter designed by the Sallen and Key method and one designed using an NIC and the Horowitz optimization technique.

The sensitivity of the transfer function to variation in gain will then be compared to that of the function designed using the NIC and optimum pole locations. A possibility of reducing the sensitivity will then be considered.

III ANALYSIS

Before postulating the basic circuit to be analyzed, it is well advised to consider the active element which is to be employed. Unlike the vacuum tube, which is a voltage amplifier, the transistor is a current-operated device which is ideally represented by a current amplifier. The circuit below is the equivalent circuit for either a current or voltage amplifier depending upon which terminal-pair is considered the input, and which the output. If E_1 is the input then the circuit is a voltage amplifier with an ideal output of $g_{21}\ E_1$. Conversely, with I_2 considered as the input, the ideal output is $g_{12}\ I_2$. The impedances g_{11} and g_{22} take into account the practical input and output conditions.

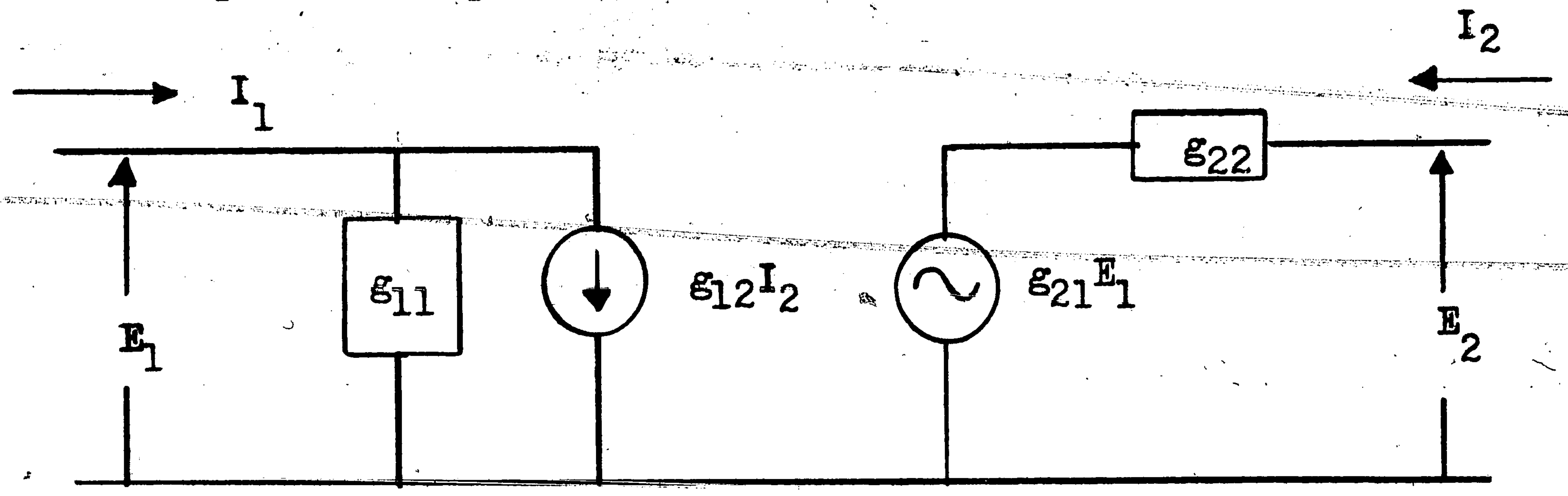


FIGURE 2

For an ideal amplifier (voltage or current) $g_{11} = g_{22} = 0$ i.e., the input impedance is infinite and the output impedance is zero for a voltage amplifier.

For vacuum tubes, the input impedance can always be made very large therefore this portion of the circuit is considered ideal. The output impedance can usually be neglected or in some cases any deviation from the ideal can be compensated for. Thus, the output circuit is also considered ideal although it is reviewed in a specific circuit to insure the validity of the assumption. The output circuit of a transistor is given the same type of inspection but again it is usually assumed that the impedance is infinite. The input impedance, however, can rarely be neglected. It is often a function of the amplifier design and should therefore be considered.

With the above discussion in mind, the circuit in Figure 3 is proposed as a general active network (with a current amplifier) to be used to realize any second order function. This circuit is not the only method of using a transistor amplifier with a feedback loop. For example, if a transistor amplifier is built which has the characteristics of an ideal voltage amplifier it could certainly be used in place of the vacuum-tube amplifier in the Sallen and Key designs.

Notice that in Figure 3 the gain K is a current gain, not a voltage gain. Also, Y_3 represents the input admittance to the amplifier and cannot be combined with Y_2 to form a single admittance. Since the analysis will be performed to obtain a current transfer ratio, it is necessary to include the load impedance.

Figure 3(b) represents the circuit with the current amplifier included and with a constant current input. The node equations for the circuit are shown below:

$$I_1 = (e_1 - e_0)\, Y_4 + (e_1 - e_2)\, Y_1$$

$$KI = (e_0 - e_1)\, Y_4 + e_0\, Y_L$$

$$0 = (e_2 - e_1)\, Y_1 + e_2\, (Y_2 + Y_3)$$

$$I = e_2\, Y_3$$

A reduction of these equations will yield the current transfer ratio

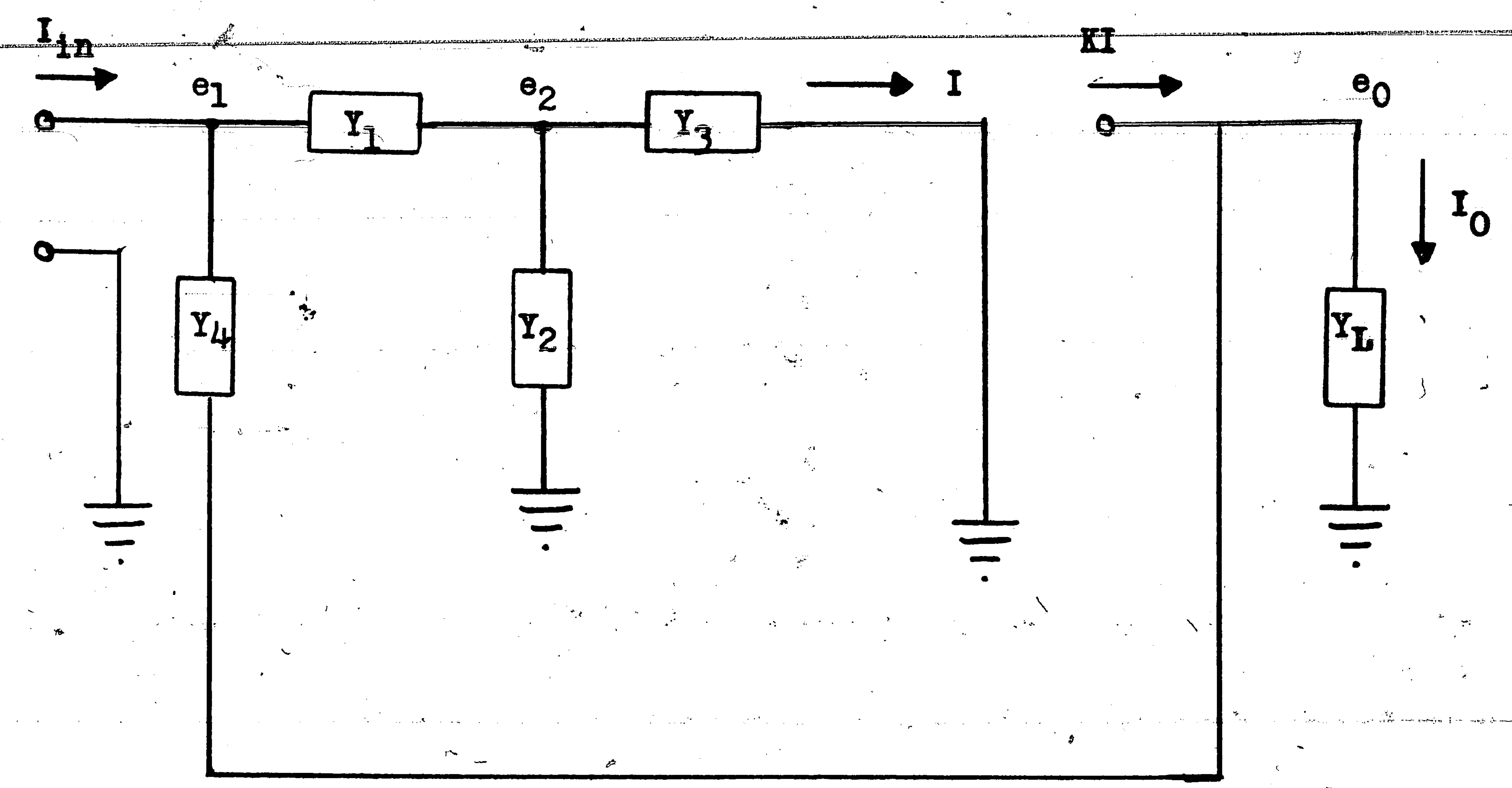


FIGURE 3(a)

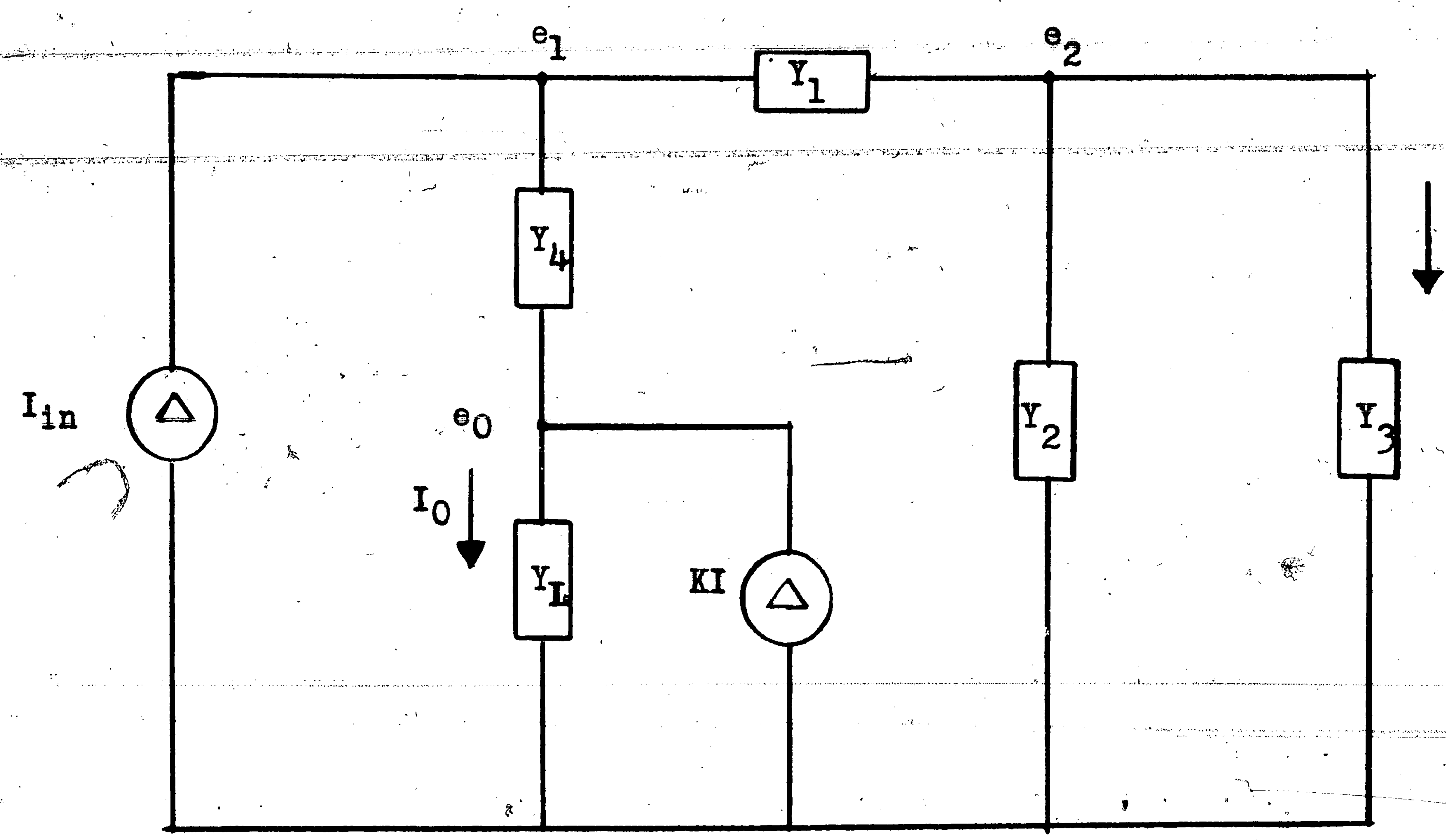


FIGURE 3(b)

$G(s) = I_L/I_1 = e_0 Y_L/I_1$ with little difficulty. The result is

$$G(s) = \frac{Y_L\left[Y_4(Y_1 + Y_2 + Y_3) + KY_1Y_3\right]}{Y_4\left[Y_1(Y_2 + Y_3) + Y_L\ (Y_1+Y_2+Y_3)\right] + Y_1Y_L(Y_2+Y_3) - KY_1Y_3Y_4}$$

where the general admittance terms are to be replaced by specific admittances to obtain the various second order functions desired.

As an example, consider the case for realizing a function of the (normalized) form

$$G(s) = \frac{hs^2}{s^2 + ds + 1}.$$

By inspection of the general function G(s) it is observed that the gain K can be used to eliminate an unwanted zero in the numerator. Let us attempt the following substitutions

$$Y_2 = sC_2 \qquad Y_1 = 1/R_1 = G_1 \qquad Y_4 = 1/R_4 = G_4$$

$$Y_L = sC_L \qquad Y_3 = 1/R_3 = G_3$$

Then

$$G(s) = \frac{sC_L\left[sC_2G_4 + G_4\ (G_1 + G_3) + KG_1G_3\right]}{s^2\left[C_2C_L(G_1+G_4)\right] + s\left[C_2G_1G_4+C_L(G_1G_4+G_3G_4+G_1G_3)\right] + G_1G_3G_4(1-K)}$$

Dividing by $G_1G_3G_4\ (1-K)$ and replacing G's with R's

$$\text{(1)} \qquad G(s) = \frac{\frac{sC_L}{1-K}\left[sC_2R_1R_3 + \ (R_1+ R_3) + KR_4\right]}{s^2\left[\frac{C_2C_LR_3(R_1+R_4)}{1 - K}\right] + \frac{s}{1-K}\left[C_2R_3+ C_L(R_1+R_3+R_4)\right] + 1}$$

If $K = -\frac{R_1+R_3}{R_4}$ the undesired factor in the numerator is eliminated.

Changing the sign of K in G(s) and normalizing by letting $C_2C_LR_3\ (R_1 + R_4) = 1 + K$, we have

$$G(s) = \frac{\dfrac{C_2C_LR_1R_3}{1 + K}\ s^2}{s^2 + \left[C_2R_3 + C_L(R_1 + R_3 + R_4)\right]\dfrac{s}{1+K} + 1}$$

which is the required result.

This illustrates the type of procedure used to obtain all the second order functions. They have all been realized and the results are available in Table I along with any conditions which must be satisfied to obtain the particular form. It can be shown (Appendix I) that the additional conditions (for eliminating zeroes) do not limit the procedure for realizing d.

Since all functions were obtained, the crux of the problem is now to obtain component values to yield a particular d. Fortunately, in every case the form of d can be shown equivalent to one derived by Sallen and Key. Therefore, their design procedure can be used. This equivalency is shown for the above example. Define

$$R_A = R_1 + R_4 \qquad T_1 = R_3C_2 \qquad A = R_3/R_A$$

$$T_2 = R_AC_L \qquad B = C_L/C_2$$

then $C_2C_LR_3\ (R_1 + R_4) = 1 + K$

$$C_2C_LR_3R_A = 1 + K$$

$$T_1T_2 = 1 + K$$

TABLE I

$$\frac{h}{s^2 + ds + 1}$$

	-K		Form
$Y_1 = sC_1$	$T_1T_2(1 + K) = 1$	T_1, B	IX
$Y_2 = G_2$	$T_1 = R_2C_3$	T_2, B	X
$Y_3 = sC_3$	$T_2 = R_LC_A$	T_2, A	IX
$Y_4 = sC_4$	$C_A = \frac{C_1C_4}{C_1+C_4}$	T_1, A	X
$Y_L = G_L$	$K = \frac{C_4(C_1+C_3)}{C_1C_3}$		
	$h = \frac{C_4}{C_1+C_4}$		

$$\frac{hs}{s^2 + ds + 1}$$

	-K		
$Y_1 = G_1$	$T_1T_2 = 1$	T_1, B	IV
$Y_2 = G_2$	$T_1 = R_2C_3$	T_2, B	II
$Y_3 = sC_3$	$T_2 = R_AC_L$	T_2, A	I
$Y_4 = G_4$	$R_A = R_1 + R_4$	T_1, A	III
$Y_L = sC_L$	$K = R_1/R_4$		
	$h = C_L(R_1 + R_2)$		

TABLE I (Cont.)

$$\frac{hs^2}{s^2 + ds + 1}$$

	$-K$		
$Y_1 = G_1$	$T_1 T_2 = 1+K$		
$Y_2 = sC_2$	$T_1 = R_3 C_2$	T_1, B	VII
$Y_3 = G_3$	$T_2 = R_A C_L$	T_2, B	VIII
$Y_4 = G_4$	$R_A = R_1 + R_4$	T_2, A	VII
$Y_L = sC_L$	$K = \frac{R_1 + R_3}{R_4}$	T_1, A	VIII
	$h = \frac{R_1 R_3 C_2 C_L}{1 + K}$		

$$\frac{h\left[s + \left(\frac{1}{\lambda}\right)\right]}{s^2 + ds + 1}$$

	$+K$		
$Y_1 = sC_1$	$T_1 T_2 = 1$	T_1, B	VI
$Y_2 = sC_2$	$T_1 = C_1 R_A$	T_2, B	V
$Y_3 = G_3$	$T_2 = C_2 R_3$	T_2, A	VI
$Y_4 = G_4$	$R_A = R_4 + R_L$	T_1, A	V
$Y_L = G_L$	$R_L = R_3$		
	$h = \lambda = R_3(C_1 + C_2) + KC_1 R_4$		

TABLE I (Cont.)

$$\frac{hs\left[s + (1/\lambda)\right]}{s^2 + ds + 1}$$

+K

$Y_1 = G_1$	$T_1T_2 = 1-K$		
$Y_2 = sC_2$	$T_1 = R_3C_2$	T_1, B	VII
$Y_3 = G_3$	$T_2 = R_AC_L$	T_2, B	VIII
$Y_4 = G_4$	$R_A = R_1 + R_4$	T_2, A	VII
$Y_L = sC_L$		T_1, A	VIII

$$h = \frac{R_1R_3C_2C_L}{1 - K}$$

$$\lambda = \frac{R_1R_3C_2}{R_1 + R_3 + K}$$

$$h \frac{s^2 + bs + 1}{s^2 + ds + 1}$$

+K

$Y_1 = G_1$	$T_1T_2 = 1$	T_1, B	V
$Y_2 = sC_2$	$T_1 = C_2R_3$	T_2, B	VI
$Y_3 = G_3$	$T_2 = C_4R_A$	T_2, A	V
$Y_4 = sC_4$	$R_A = R_1 + R_L$	T_1, A	VI
$Y_L = G_L$	$R_L = R_3$		

$$s^2 + bs + 1 = s^2T_1R_1C_4 + sT_2 + K$$

$$h = 1$$

And

$$d = \frac{1}{1+K}\left[C_2R_3 + C_L(R_1 + R_4) + C_LR_3\right]$$

$$= \frac{1}{1+K}\left[T_1 + T_2 + AT_1\right]$$

Or

$$= \frac{1}{1+K}\left[T_1 + T_2 + BT_2\right].$$

The equation can now be written

$$d = \frac{T_1\ (1 + B)}{1 + K} + \frac{1}{T_1} = \frac{T_2(1 + A)}{1 + K} + \frac{1}{T_2}$$

or

$$d = \frac{1 + B}{T_2} + \frac{T_2}{1+K} = \frac{1 + A}{T_1} + \frac{T_1}{1+K}$$

These are forms VII and VIII respectively in the Sallen and Key tabulation. (Table II is a reproduction of all the design formulae)

An examination of Table I indicates that in a number of cases the load impedance is capacative. Therefore, when using these forms, it should be kept in mind that, at low frequencies, the output impedance of the amplifier may no longer be considered infinite. This will cause reduced rejection at these frequencies.

It should be noticed that the use of five components allows some freedom of selection since two pairs of components must be combined to achieve two time constants. In some cases the freedom must be used to eliminate the undesired zero; in other cases the components should be selected to reduce the pass band loss.

TABLE II DESIGN FORMULAS FOR DISSIPATION FACTOR d

(This Table has been extracted from the Sallen and Key article, reference #6)

GROUP	(a) $d(x,K,T)$	(b) $T(x,K,d)$	(c) $K(x,d,T)$	(d) K_{min}	(e) x_{min}	(f) $T_{K_{min}}$	(g) K_{max}
I	$\frac{(1-K)}{T}+T(1+x)$	$\frac{d}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)(1-K)}{d^2}}\right]$	$T^2(1+x)+1-dT$	$\frac{4(1+x)-d^2}{4(1+x)}(<1)$	$\frac{d^2-4(1-K)}{4(1-K)}$	$\frac{d}{2(1+x)}$	$T^2(1+x)+1$
II	$\frac{(1+x-K)}{T}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x-K)}{d^2}}\right]$	$T^2+(1+x)-dT$	$\frac{4(1+x)-d^2}{4}$	$\frac{d^2-4(1-K)}{4}$	$\frac{d}{2}$	$T^2+(1+x)$
III	$\frac{(1+x)}{T}+T(1-K)$	$\frac{d}{2(1-K)}\left[1\pm\sqrt{1-\frac{4(1+x)(1-K)}{d^2}}\right]$	$\frac{T^2+(1+x)-dT}{T^2}$	$\frac{4(1+x)-d^2}{4(1+x)}(<1)$	$\frac{d^2-4(1-K)}{4(1-K)}$	$\frac{2(1+x)}{d}$	$\frac{T^2+(1+x)}{T^2}$
IV	$\frac{1}{T}+T(1+x-K)$	$\frac{d}{2(1+x-K)}\left[1\pm\sqrt{1-\frac{4(1+x-K)}{d^2}}\right]$	$\frac{T^2(1+x)+1-dT}{T^2}$	$\frac{4(1+x)-d^2}{4}$	$\frac{d^2-4(1-K)}{4}$	$\frac{2}{d}$	$\frac{T^2(1+x)+1}{T^2}$
V	$\frac{1}{T}+T[1+x(1-K)]$	$\frac{d}{2[1+x(1-K)]}\left\{1\pm\sqrt{1-\frac{4[1+x(1-K)]}{d^2}}\right\}$	$\frac{T^2(1+x)+1-dT}{xT^2}$	$\frac{4(1+x)-d^2}{4x}(>1)$	$\frac{4-d^2}{4(K-1)}$	$\frac{2}{d}$	$\frac{T^2(1+x)+1}{xT^2}$
VI	$\frac{[1+x(1-K)]}{T}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{x}$	$\frac{4(1+x)-d^2}{4x}(>1)$	$\frac{4-d^2}{4(K-1)}$	$\frac{d}{2}$	$\frac{T^2+(1+x)}{x}$
VII	$\frac{1}{T}+\frac{T(1+x)}{(1+K)}$	$\frac{d(1+K)}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2(1+x)+1-dT}{(dT-1)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{2}{d}$	Infinite
VIII	$\frac{(1+x)}{T}+\frac{T}{(1+K)}$	$\frac{d(1+K)}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{dT-(1+x)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{2(1+x)}{d}$	Infinite
IX	$\frac{1}{T(1+K)}+T(1+x)$	$\frac{d}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2(1+x)+1-dT}{dT-T^2(1+x)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{d}{2(1+x)}$	Infinite
X	$\frac{(1+x)}{T(1+K)}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{dT-T^2}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{d}{2}$	Infinite

$T = T_1$ or T_2, as appropriate; $x = A$ or B, as appropriate.

IV DESIGN PROCEDURE

The realization of a required d is identical to that used by Sallen and Key. However, a brief review of the method will be presented.

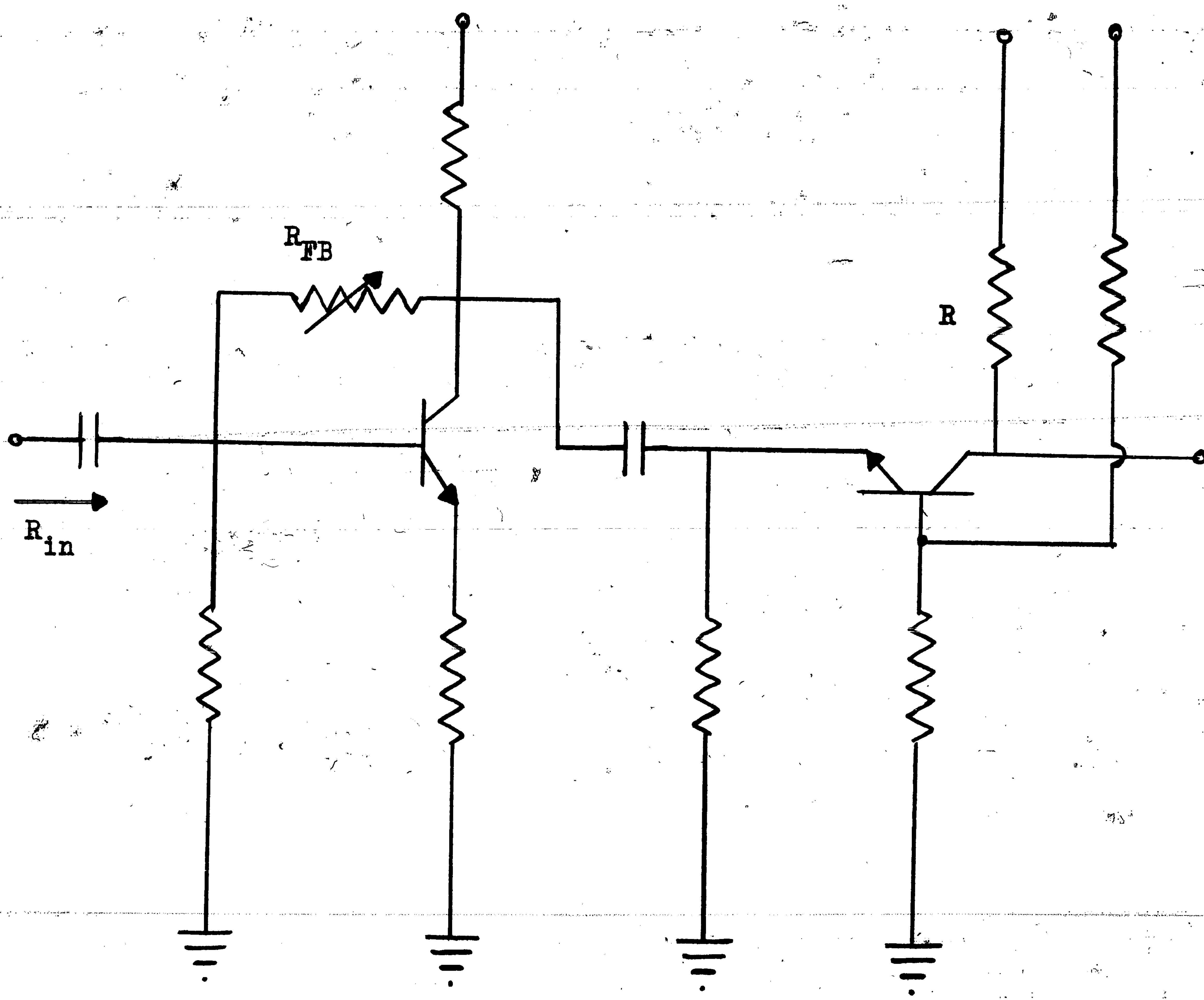
Given a transfer function, a circuit to realize it is available in Table I. This table also contains four forms for obtaining the desired value of d. No matter which form is selected, three variables appear, K, X, T. X can be either the ratio of resistors or of capacitors and T is either time constant (both as shown). Theoretically, two can be arbitrarily chosen. Practically, an X is usually selected to obtain a range of values for the gain K. When K is selected all the parameters (T_1, T_2, X, K) are fixed by the design equations shown. By satisfying any additional conditions or by minimizing pass band losses the component values can all be obtained.

After the five component values have been realized, then it is necessary to build the current amplifier. This should be designed with the following criteria as guides:

1. The input impedance must be equal to the value calculated, (Y_3).
2. The circuit must have the overall current gain required.
3. The output admittance must be as large as possible.
4. The amplifier must be stable.
5. The feedback voltage ratio E_{IN}/E_{OUT} must be as small as possible.

Violation of any of the criteria will reduce the effectiveness of the filter.

The amplifier design used to realize the Butterworth filter is shown in Figure 4. The input resistance can be adjusted to satisfy Y_3 and the feedback resistance R_{fb} can be used to obtain the desired current gain. The biasing resistor R should be made as large as possible to satisfy the ideal output conditions.



Current Amplifier

FIGURE 4

V SENSITIVITY CONSIDERATIONS

The possible variation in gain of a simple transistor amplifier is much greater than a simple vacuum-tube amplifier. Transistor parameters have a wide spread making inter-changeability a problem; and some parameters show ageing effects and most are sensitive to temperature variation. Since the gain is usually affected by such variations, it is of interest to define the variation of certain parameters with respect to gain.

One method of measuring this sensitivity is to find the ratio of the percentage change in the parameter to the percentage change in the gain.

$$S_K^P = \frac{\partial P/P}{\partial K/K} = \frac{\partial P}{\partial K} \cdot \frac{K}{P}$$

It is of course desirable to minimize the sensitivity.

In the previous example, minimizing the variation of both poles and the zero will minimize the total sensitivity of G(s) with respect to K. Writing the function in somewhat different form we have:

$$G(s) = \frac{hs\,(s+\alpha)}{s^2 + A_1 s + A_2}$$

where

$$h = \frac{R_1}{R_1 + R_4}$$

$$\alpha = \frac{R_1 + R_3 - KR_4}{R_1 R_3 C_2}$$

$$A_1 = \frac{1}{C_L(R_1 + R_4)} + \frac{1}{C_2(R_1 + R_4)}$$

$$A_2 = \frac{1 + K}{C_2 C_L R_3 (R_1 + R_4)}$$

$$s^2 + A_1 s + A_2 = (s + a)\ (s + b)$$

$$a = -\frac{A_1}{2} + 1/2\sqrt{A_1^{\ 2} - 4A_2}$$

$$b = -\frac{A_1}{2} - 1/2\sqrt{A_1^{\ 2} - 4A_2}$$

The sensitivity of the singularities are given below. They are always given in absolute value.

$$\frac{\partial a}{\partial K} \cdot \frac{K}{a} = \frac{2b}{2K} \cdot \frac{K}{b} = S_K^{a,b}$$

$$S_K^{a,b} = \frac{1}{b(2b + A_1)} \cdot \frac{K}{1 + K}$$

$$S_K^{\alpha} = \frac{B\left[T_1^{\ 2}B + 1 + K\right]}{T_1\left[B(T_1^{\ 2}B + 1 + K) - T_1^{\ 2}C_L^{\ 2}(1+K)\right]}$$

To minimize the sensitivity of the poles it is necessary to obtain the smallest possible gain. Theoretically, the gain can assume any value greater than one. For this form therefore, the circuit would be designed with a gain as near one as possible. However, as the circuit gain is reduced, the circuit elements must be changed and may become physically large and this is usually the limiting design factor.

The sensitivity of the zero location is reduced by selecting as large an output capacitance as possible, leading to another limit based on component size. This is normally of little concern, however, since the zero usually lies well out of the pass band.

Sensitivity figures, per se, are not too significant unless they can be compared to some reference. This is provided by Linvill's NIC method employing Horowitz' optimum decomposition technique. Although the method of minimizing the sensitivity of each pole and zero was used to minimize the sensitivity of the function, this is not practical for comparing two methods. The direct approach is to compare the sensitivity of the transfer function, G(s).

The equation for sensitivity using the transistor amplifier is

(2)

$$S_K^{G(s)} = \frac{K\sqrt{R_4^{\,2}\left\{(1+K) - \omega^2 C_2 C_L R_3 R_A\right\}^2 + \omega^2\left\{R_1 R_3 C_2 + R_4\left[C_2 R_3 + C_L(R_1+R_3+R_4)\right]\right\}^2}}{\omega C_2 R_1 R_3\sqrt{\left\{(1+K) - \omega^2 C_2 C_L R_3 R_A\right\}^2 + \omega^2\left\{C_2 R_3 + C_L(R_1+R_3+R_4)\right\}^2}}$$

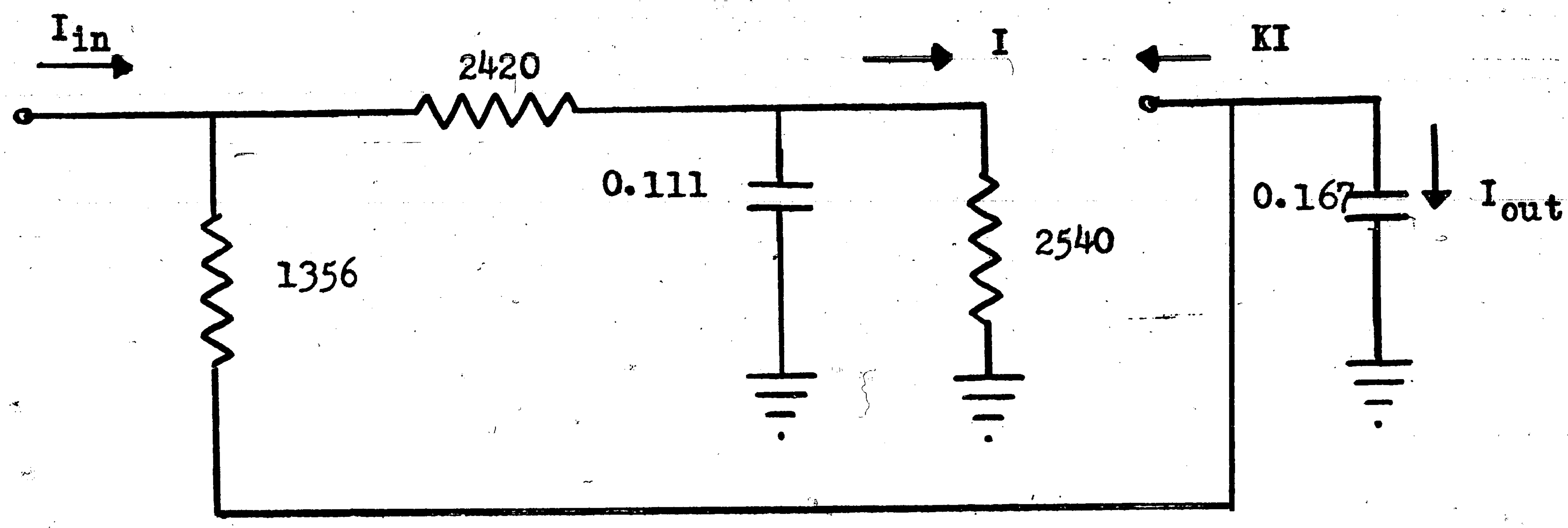
If the Linvill-Horowitz approach is used to design the second-order function previously mentioned, the sensitivity is *

(3)

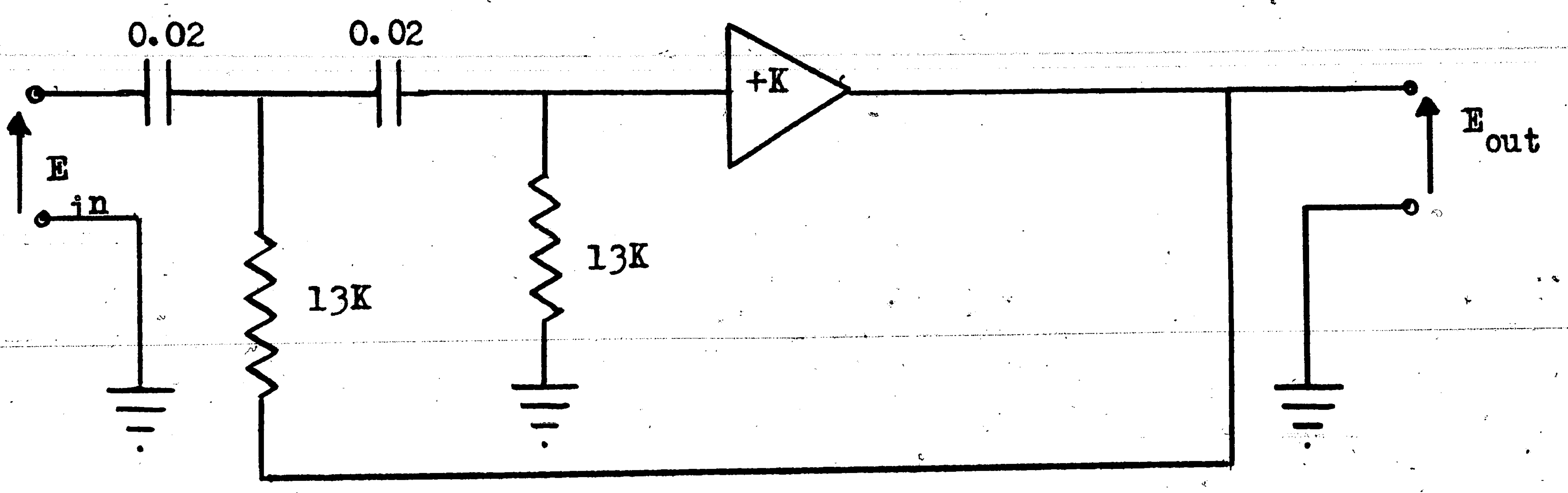
$$S_K^{G(s)} = \frac{K\omega C_1 R_2\sqrt{\left[C_2 R_3 \omega\right]^2 - 1}}{\sqrt{\left\{1-\omega^2\left[C_1 C_2 R_1(R_2+R_3) - KC_1 C_2 R_2 R_3\right]\right\}^2 + \omega^2\left\{C_1 R_1 + C_2(R_2+R_3) - KC_1 R_2\right\}^2}}$$

The circuit which has this sensitivity function is shown in Figure 5.

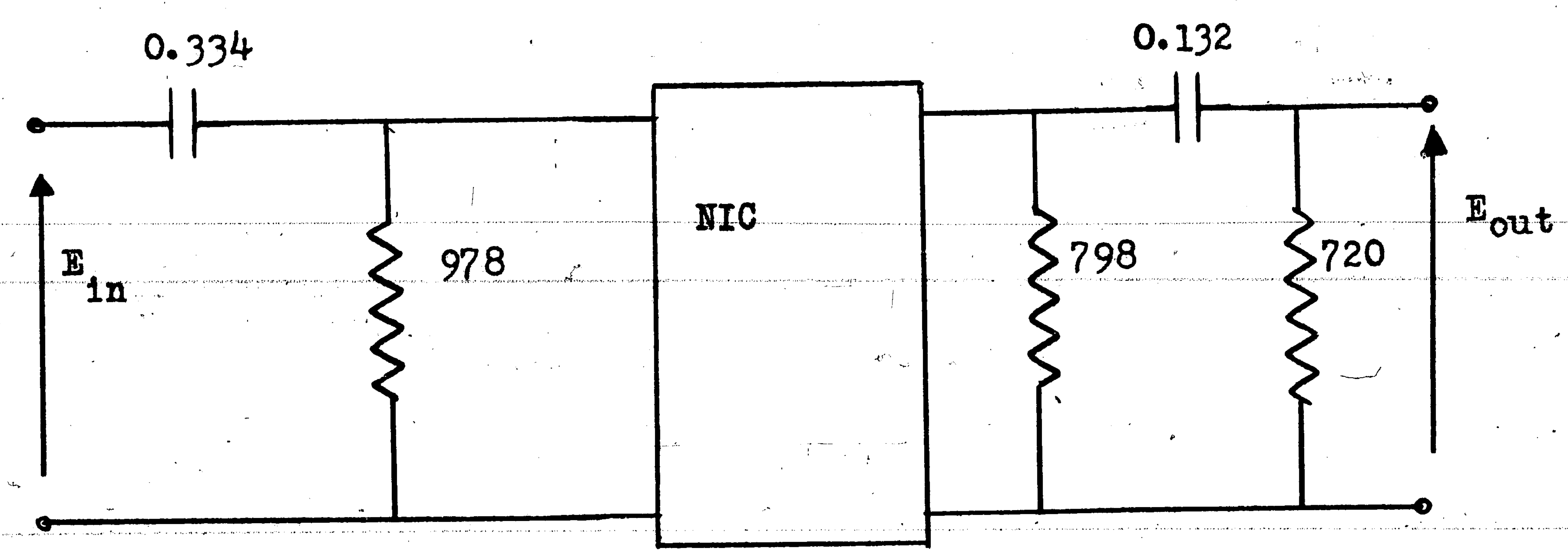
* See Appendix III.



TRANSISTOR CIRCUIT K = 4



VACUUM-TUBE CIRCUIT K = 0.75



OPTIMUM NIC CIRCUIT

FIGURE 5

VI EXPERIMENTAL VERIFICATION

To ensure that the preceding theoretical work has some practical application, a transfer function with a Butterworth characteristic namely

$$G(s) = \frac{hs^2}{s^2 + 1.414\,s + 1}$$

was chosen to be realized. The numerical computations for this high pass filter are shown in Appendix II. The gain was arbitrarily chosen to be four and no attempt was made to optimize the sensitivity of the design. The circuit is designed for an impedance level of 600 ohms and to have a cutoff of 5000 radians per second. The response curve is shown in Figure 6.

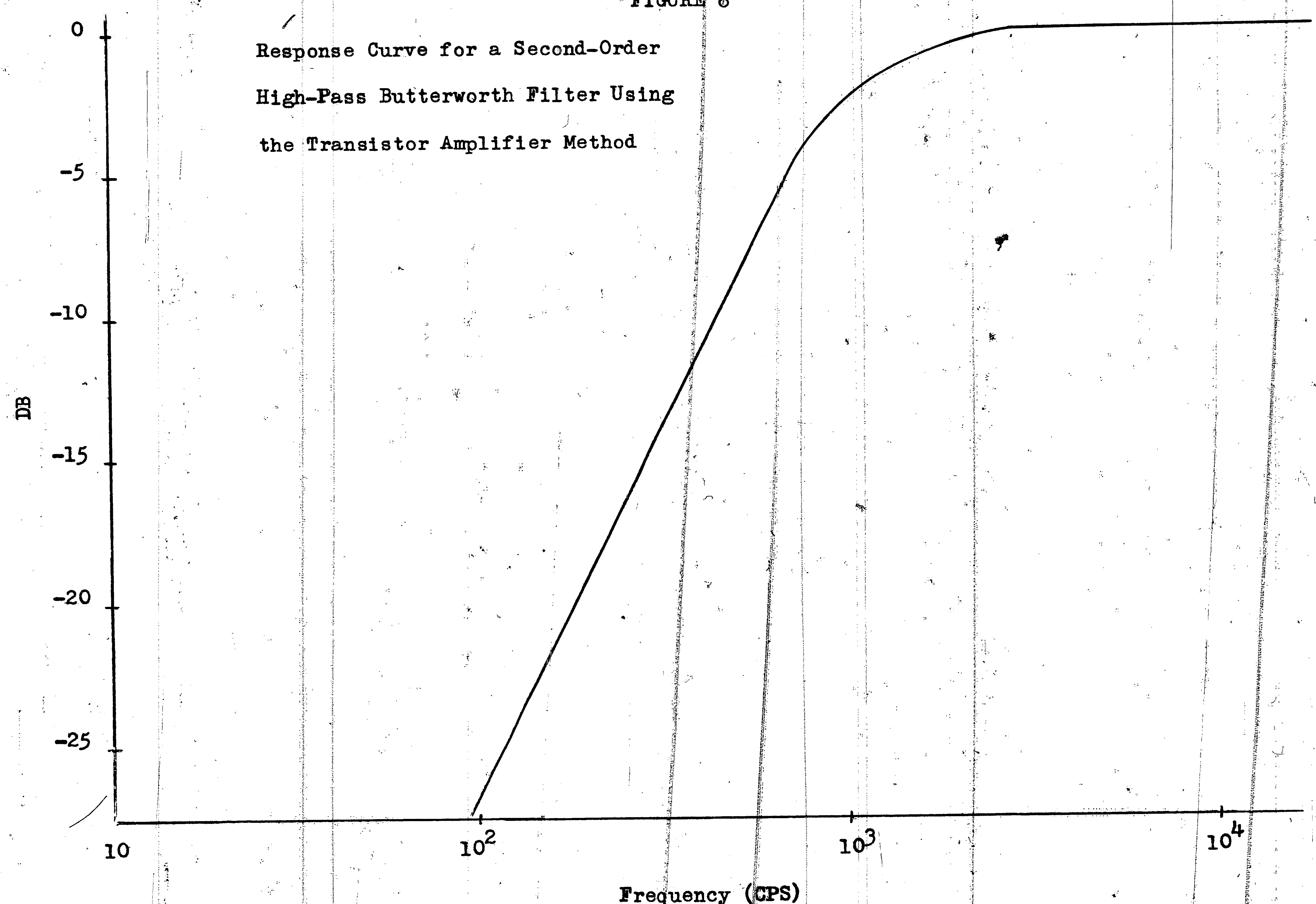
As a basis for comparison, the same filter was designed using a vacuum-tube circuit (Sallen and Key method) and using an NIC including optimizing.

The results of the calculations for the latter circuit are also shown in Appendix II. The response data obtained by using each of the three methods is presented in Table III, and the circuits used are shown in Figure 5.

An examination of the tabulated results shows that the response in each case is very similar. The assumptions made for the various circuits lead to the deviation from the ideal 12 DB per octave rejection. In the case of the transistor amplifier, the finite output impedance is the major cause of deviation from the ideal. This could be improved by improving the amplifier design.

Since the design technique has been verified, a new circuit was designed to optimize the sensitivity. The design results are also shown

FIGURE 6



Response Curve for a Second-Order High-Pass Butterworth Filter Using the Transistor Amplifier Method

TABLE III

Characteristic Curve Data for Second Order

Butterworth Function

1. NIC Method

2. Sallen and Key Method

3. Current Amplifier

	NIC	S and K	TRANSISTOR
f(CPS)	|G(s)|(DB)		
100	30.2	28.0	26.4
200	20.0	18.0	18.9
300	14.3	13.0	15.2
400	10.2	9.8	11.3
700	5.1	4.8	5.6
800	4.0	4.0	4.0
900	3.3	3.3	3.4
1000	2.7	2.8	2.9
1500	0.9	1.5	1.3
2000	0.7	0.8	0.4
4000	0.0	0.0	0.0
10,000	0.0	0.0	0.0

in Appendix II. Optimizing was constrained by limits chosen for element values. The limits arbitrarily selected were:

1. No capacitor shall exceed ten microfarads.
2. No resistor shall be less than 150 ohms.

Using these criteria, the gain was reduced from 4 to 1.2 with a corresponding reduction in sensitivity. As a comparison, the sensitivity of the NIC design above was calculated and compared to that of the transistor designs for gains of 4 to 1.2. The results of these calculations using equations 1 and 2 in the previous section are shown in Table IV and in Figure 7.

As can be seen from the plot of sensitivity vs frequency, the transistor amplifier is less sensitive to changes in gain at frequencies below 250 cps., however, above this frequency the sensitivity rises rapidly. Since the design cutoff frequency is 800 cps. this sensitivity compares very favorably with that of the NIC.

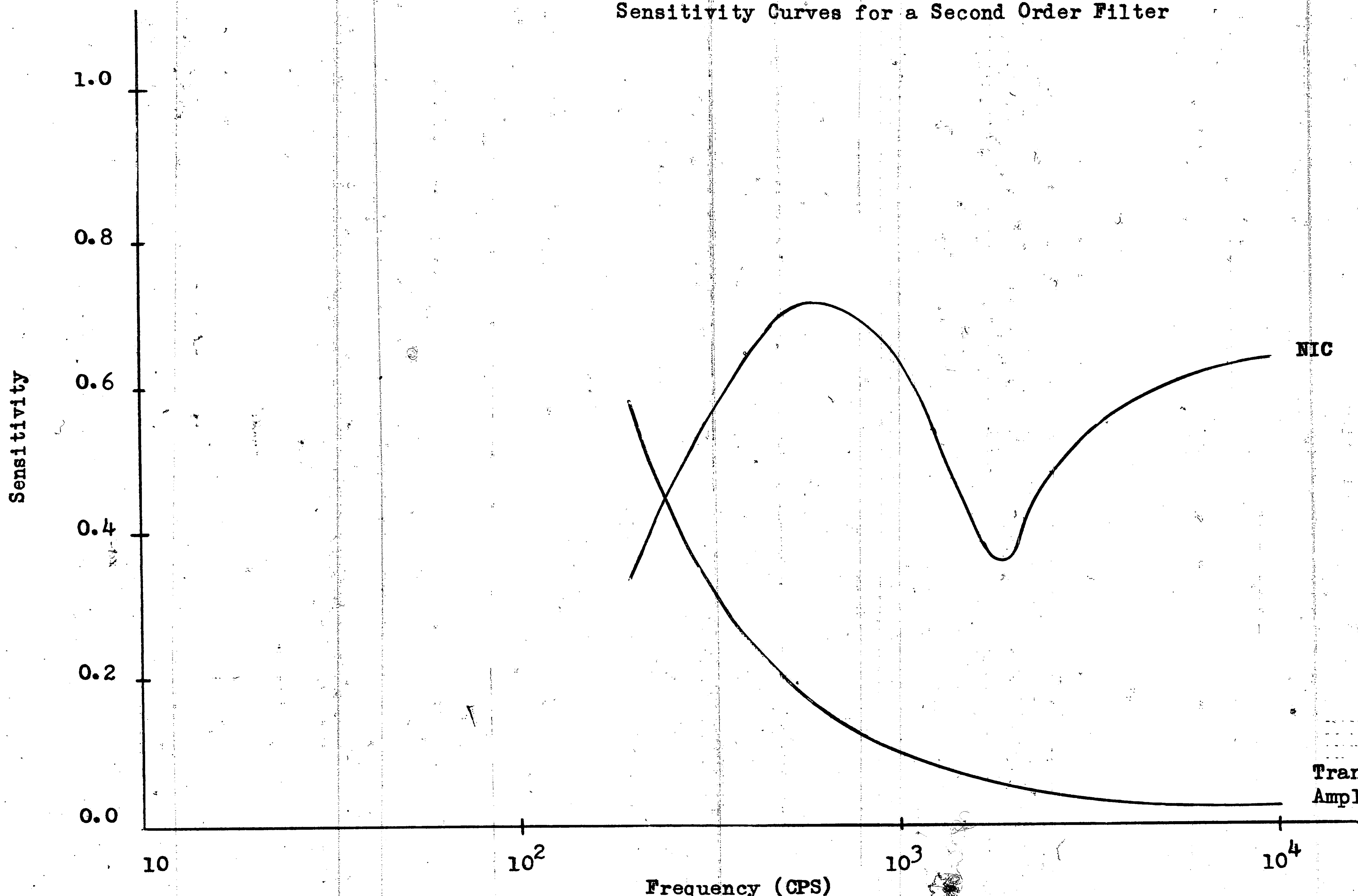
TABLE IV

Calculated Sensitivity for NIC and Transistor Amplifier with Feedback

	NIC	TRANSISTOR AMPLIFIER	
f	S	S(K = 1.2)	S (K = 4.0)
200	0.334	0.580	4.210
300	0.503	0.346	3.520
400	0.652	0.253	2.760
800	0.717	0.122	1.630
2,000	0.347	0.048	0.498
5,000	0.595	0.019	0.139

FIGURE 7

Sensitivity Curves for a Second Order Filter



VII COMPARISON OF DESIGNS

Since the design technique for the transistor circuit is basically the same as the vacuum-tube circuit there is little choice between the two. However, the deviation of the input impedance from the ideal for the transistor places an additional component into the design procedure. The fifth component actually inserts a degree of freedom. This is used either to eliminate an undesired zero from the numerator of the transfer function or to decrease the pass band loss. In the first case, all components are explicitly defined. In the latter case, the components can be selected with a degree of freedom. In all cases, however, five major passive components are required for the transistor circuit rather than four as in the vacuum-tube circuit.

The use of the smaller circuit with less power dissipation is certainly an advantage, but there is a drawback. In all the vacuum-tube circuits presented by Sallen and Key, a one stage amplifier could be used. Unfortunately with the transistor circuits very often (with capacitive loads) two stages must be employed, one for amplification and one for isolation.

When the transistor circuit is compared to the NIC circuit, the sensitivities do not give either approach a distinct advantage. Also, the number of passive components and transistors are identical. However, since the transistor amplifier requires a minimum amount of adjustment after construction, it has a considerable advantage over the NIC.

VIII CONCLUSION

It was not unexpected that second-order functions could be realized using a transistor amplifier. However, the introduction of an additional passive design component could have complicated the problem to the point of making the method useless. This fortunately was not the case; any additional conditions imposed are readily handled. In fact, the design procedure for realizing any second-order function is a straightforward one, the basic technique of which has been used before.

After the five component values are determined, the amplifier must be designed to give the proper input impedance, high output admittance and stable gain. The latter is not difficult to obtain since the gain is to be kept as low as possible. A stable circuit is easily obtainable and there is a minimum of adjustment required after construction.

The input impedance is easily adjusted to any desired value although if a capacitive input is required, a minimum resistance of approximately 25 ohms will appear in series with it. If other resistive values are kept large, in comparison, the effect can be neglected. The output admittance presents a more difficult problem if the load is capacitive. The only solution is to design the amplifier to minimize its effect.

If component sizes are not overly restricted, the sensitivity of a circuit can be considerably reduced. This is demonstrated by the results shown in Table IV. This table also indicates the favorable comparison of sensitivity using the transistor or NIC.

The characteristic response curve resulting from this design technique can be favorably compared to the vacuum-tube circuit and to the NIC. The results obtained, together with the ease of design method and adjustment certainly make this approach to RC Active Filter design a practical and useful one.

IX APPENDIX I

The purpose of this section is to show that the additional conditions which must be imposed on K do not prevent realization of the form. In doing so a method for realizing the components is shown.

(1) $$G(s) = \frac{h}{s^2 + ds + 1}$$

$$K = \frac{C_4(C_1 + C_3)}{C_1C_3} \qquad C_A = \frac{C_1C_4}{C_1 + C_4}$$

$$C_4(C_1 + C_3) - KC_1C_3 = 0 \qquad C_A(C_1 + C_4) = C_1C_4$$

Solve the above equations for C_1. The result is

$$C_1 = \frac{(1 + K)\ C_3C_A}{K\ C_3 - C_A}$$

$$C_1 = \frac{C_A(1 + K)}{K - B}$$

Note: $K > B$ is the only necessary condition. Since K can have any value up to infinity this is no problem. C_1 is always larger than C_A so C_4 is not unattainable.

(2)

$$G(s) = \frac{hs}{s^2 + ds + 1}$$

$$K = \frac{R_1}{R_4} \qquad R_A = R_1 + R_4$$

The sum can always be selected to realize the ratio.

(3) $$G(s) = \frac{hs^2}{s^2 + ds + 1}$$

$$K = \frac{R_1 + R_3}{R_4} \qquad\qquad R_A = R_1 + R_4$$

In this case K must always be larger than 1 so again it is possible to select the sum $(R_1 + R_4)$ to yield the correct ratio.

In both of these last two cases the time constants determine the exact component values.

APPENDIX II

The design procedure for realizing the transfer function

$$G(s) = \frac{hs^2}{s^2 + 1.414\ s + 1}$$

is shown here. The first case is an arbitrarily chosen K and the second choses K to minimize the sensitivity and yet limit component to $C \leqslant 10$ uf and $R \geqslant 150$ ohms.

(1) Select $B = 3/2$ (using Form VIII, Table II)

$$K\text{min} = \frac{4(\ 1 + B) - d^2}{d^2}$$

The range of values for K is 4 to ∞. Therefore, arbitrarily chosing 4 we have

$$T_2 = \frac{d(1 + K)}{2}\left[1 \pm \sqrt{1 - \frac{4(1 + K)}{d^2(1 + K)}}\right]$$

$$T_2 = 3.54$$

$$T_1 = \frac{1 + K}{T_2}$$

$$T_1 = 1.41$$

Let $\quad C_L = 1/2 \quad$ and $C_2 = 1/3$

Then $\quad T_1 = C_2R_3 \qquad T_2 = R_AC_L$

$$R_3 = 4.23 \qquad R_A = 7.08$$

From the additional condition that

$$K = \frac{R_1 + R_3}{R_4}$$

$$4R_4 = R_1 + 4.23$$

Also $R_A = R_1 + R_4 = 7.08$

Solving simultaneously

$$R_4 = 2.26 \qquad R_1 = 4.02$$

The final values both actual and normalized are shown below. The actual is based on $\omega_0 = 5000$ radius/second and an impedance level of 600 ohms.

	NORMALIZED	ACTUAL	
R_1	4.020	2420	OHMS
R_3	4.230	2540	
R_4	2.260	1356	
C_2	0.334	0.111	MICROFARADS
C_L	0.500	0.167	

(2) Following the same procedure but optimizing yields:

	NORMALIZED	ACTUAL	
R_1	3.400	2040	OHMS
R_3	0.300	180	
R_4	3.100	1860	
C_2	3.000	10	MICROFARADS
C_L	0.300	1	

For completeness, the results of the design of the NIC circuit, using Linvill's method and Horowitz' optimum pole locations, are included.

	NORMALIZED	ACTUAL	
R_1	1.630	978	OHMS
R_2	1.330	798	
R_3	1.200	720	
C_1	1.000	0.334	MICROFARADS
C_2	0.396	0.132	

APPENDIX III

To obtain the sensitivity for the optimum transistor circuit derived in this work, the partial deriviative of G(s) (Equation 1) with respect to K is multiplied by K and divided by G(s). The absolute value of this equation is then found and the result is shown in Equation 2.

The sensitivity for the Linvill-Horowitz filter is derived in a similar manner using the following equation for the transfer function.

$$G(s) = \frac{R_1R_2R_3C_2s^2}{s^2\left[C_1C_2R_1(R_2+R_3) - KC_1C_2R_2R_3\right] + s\left[C_1R_1+C_2(R_2+R_3)-KC_1R_2\right]+1}$$

The sensitivity equation is now derived to indicate the procedure used for both cases. Let D stand for the denominator of G(s) in the above relationship.

$$\frac{\partial G(s)}{\partial K}\cdot\frac{K}{G(s)} = \frac{\left\{R_1R_2R_3s^2\left[C_1C_2R_2R_3s^2 + C_1R_2s\right]\right\} KD}{D^2R_1R_2R_3C_2s^2}$$

Simplifying:

$$\frac{\partial G(s)}{\partial K}\cdot\frac{K}{G(s)} = \frac{sC_1R_2K\left[C_2R_3s + 1\right]}{\left\{s^2C_1C_2\left[R_1(R_2+R_3) - KR_2R_3\right] + s\left[C_1R_1+C_2(R_2+R_3)-KC_1R_2\right]+1\right\}}$$

Let $s = j\omega$

$$S_K^{G(s)} = \left|\frac{\partial G(s)}{\partial K}\cdot\frac{K}{G(s)}\right| = \frac{K\omega C_1R_2\sqrt{\left[C_2R_3\omega\right]^2 - 1}}{\sqrt{\left\{1-\left[C_1C_2\omega^2(R_1R_2+R_1R_3-KR_2R_3\right]\right\}^2 + \left\{\omega\left[C_1R_1+C_2(R_2+R_3)-KC_1R_2\right]\right\}^2}}$$

This is Equation 3.

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